Transitional Flow Separation Upstream of a Compression Corner

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Experimental measurements of transitional separation, caused by a compression corner on a two-dimensional model were analyzed in an attempt to correlate the extent of separation. A previously published correlation, based on the wetted length Reynolds number at the separation point, failed when the plate length alone was changed. A re-examination of the data showed that the true correlating parameter was the freestream unit Reynolds number. Examination of the equation of motion applicable to the dividing streamline between the boundary layer and the recirculating flow suggested that the unit Reynolds number determined the growth of turbulence in the transition region. The separated flow transition could then be related to attached-flow, boundary-layer transition on a plate, with the same external flow condition. A new correlation showed that the length of transitional separation was a function only of freestream Mach number, inviscid flow pressure rise at the compression corner, and the length over which transition develops in an attached-flow, flat-plate boundary layer with the same external flow conditions.

Nomenclature

L_{sep} = length of separated flow; L_{sep}', from pressure distribution, in.
ΔL = error in L
M = Mach number
p = pressure
p_2 = plateau pressure after separation
p_3 = peak pressure after reattachment
p_f = flap pressure in uniform inviscid flow
R = gas constant
Re = Reynolds number; Re_x, based on length
T = temperature
u,v = boundary-layer velocity components
x,y = Cartesian coordinates
x_h,x_s = plate lengths to hinge line and separation point
$(x_t)_{end}$ = plate length to end of boundary-layer transition
Δx_{tr} = transition length
α_{sep} = angle between dividing streamline and plate surface
δ_f = flap deflection angle
ϵ = eddy viscosity
ζ,ν = effective viscosity and kinematic viscosity, respectively
ρ = density
τ = shear stress

Subscripts

0,2 = beginning of pressure rise and separation plateau
r,w = reattachment and wall, respectively

Superscripts

$(\ )'$ = in coordinate system of dividing streamline

Introduction

FLOW separation can drastically reduce aerodynamic control effectiveness, particularly for maneuvering re-entry vehicles. At high altitudes the boundary layer tends to remain laminar, and separation, when it occurs, is extensive. The separation caused by a compression corner (see Fig. 1) may be either wholly laminar or transitional, with transition to turbulence beginning before the separated flow reattaches. To determine the effects of separation we must know its extent, and the resulting changes in pressure distribution.

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Laminar separation has been studied in detail. Transitional separation on the other hand has been almost completely neglected, both theoretically and experimentally. Thus, Chapman, Kuehn, and Larson[1] showed that their laminar separation and plateau pressure correlations could be applied to transitional separation upstream of a forward facing step under three conditions: 1) transition is not too close to separation; 2) the pressure distribution has a length of sensibly constant plateau pressure not less than about 1.5 times the length over which it takes the pressure to rise from p_0 to p_2 (see Fig. 1); and 3) the disturbance due to transition, as measured by the pressure rise over the laminar plateau pressure, must not exceed 2 to 3 times $(p_2 - p_0)$.

Larson and Keating[2] studied the influence of Mach number (M = 2.06 to 4.24), Reynolds number, and wall cooling on transition in separated flows. They used an ogive cylinder with an axisymmetric notch or cavity to show that the transition Reynolds number decreased with increasing wall cooling, increased with increasing Mach number and increased with increasing unit Reynolds number $(Re/\text{in.})_0$.

Needham and Stollery[3] correlated separation length for laminar, transitional, and turbulent boundary layers on a flat plate with a compression corner. They found that laminar separation increases as Reynolds number is increased, while transitional separation decreases. Their approach was to plot $L_{sep}M_0^3/x_0\ (p_3/p_2)^2$ vs Re_{x0}, using L_{sep} measured from schlieren photographs.

The starting point for this study was a discrepancy that appeared when the Needham-Stollery correlation was applied to new transitional separation data. The end product, presented below, is a new correlation of transitional separation in

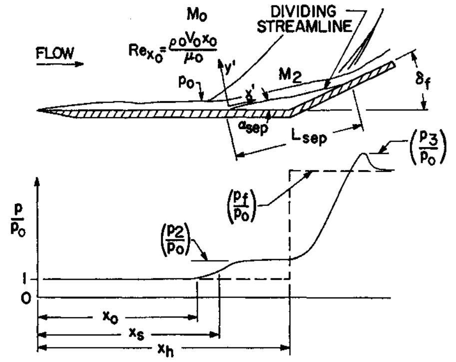

Fig. 1 Flow separation in a compression corner.

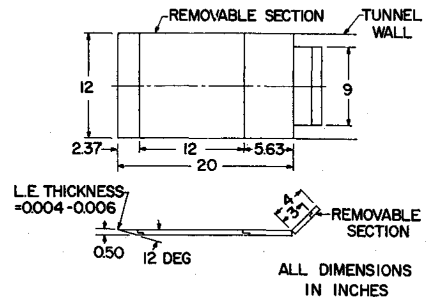

Fig. 2 Flat-plate, flow separation model used in the G.E. study.[4–7]

a compression corner. This correlation relates the transitional separation length to the length over which transition would occur in an undisturbed flow, defined as one in which the compression corner is removed and the flat plate extended indefinitely. The correlation is discussed in terms of the flow mechanism that determines transitional L_{sep} and the requirements that must be satisfied in future transitional flow separation experiments.

Experimental Data

The data used in the present General Electric (GE) study were obtained in an experimental program[4–7] conducted in Tunnel D of the Arnold Engineering Development Center (AEDC). The test model is described by Fig. 2. The length x_h could be shortened to 8 in. by removing a 12-in. section. There was no heat transfer to the model surface. Data were taken at $M = 3.0$ and 4.5, with the plate shortened to 8 in. at various values of $(Re/\text{in.})_0$ (30,000–300,000) and flap angles δ_f(7.5°–30°).

Continuous source schlieren photographs were taken, and p_w and T_w (to verify the adiabatic wall) were measured. With the plate extended to $x_h = 20$ in. and $\delta_f = 0$, the beginning and end of boundary-layer transition in the undisturbed flow were determined by means of a boundary-layer probe. The flat plate used in these tests was subsequently shortened to 3.8-in. length by J. D. Gray of AEDC (Fig. 3) and used to obtain additional Mach 3 separation data under approximately the same range of flow conditions.[8]

Separated Flowfield

In Fig. 1 the boundary layer grows undisturbed up to x_0, where the separation interaction begins. It separates at x_s, and the separating streamline makes an almost constant angle α_{sep} with the plate to the beginning of reattachment. The plateau pressure ratio p_2/p_0 is determined by α_{sep}. The flow reattaches at x_r. After reattachment the pressure rises to a peak value p_3 and then decays to p_f, the flap pressure for an inviscid flow. The parameter we wish to study is L_{sep}.

For some separated flow data[8] the pressure distribution does not show a lengthy plateau, although it does show a "knee."

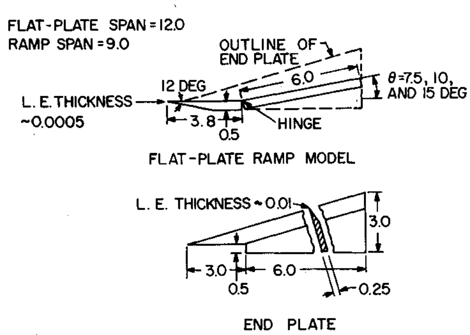

Fig. 3 AEDC flow separation model.

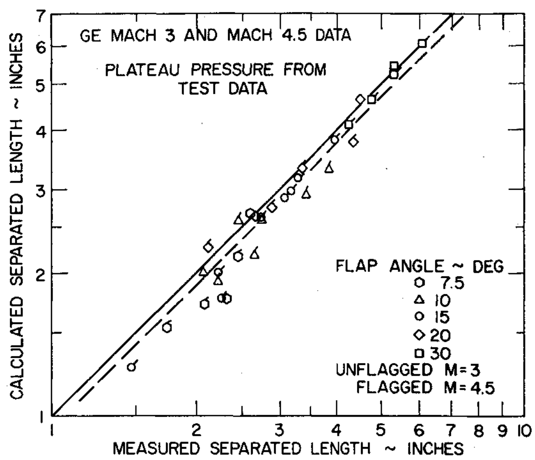

Fig. 4 Comparison of separated lengths calculated from plateau pressure with those measured from schlieren photographs.

In this case the pressure ratio at the second inflection point, where the slope is nearly horizontal, is taken as p_2/p_0.

When the boundary layer becomes fully turbulent upstream of the compression corner as observed in schlieren photographs, there is an abrupt change in the pressure distribution. A pressure gradient, intermediate between the zero gradient of the plateau region and the reattachment gradient, appears. This was reported by Chapman, Kuehn, and Larson,[1] and some cases were also observed in the GE tests[4,5]; however, this type of transitional separation data was not included in the analysis.

Length of Separated Flow

The separating streamline of Fig. 1 is almost a straight line over its whole length. By assuming it to be a straight line to reattachment we can use the schlieren photographs from the GE experimental program[4–7] to measure x_s, L_{sep}, and α_{sep}.

Unfortunately, much of the flow separation data reported in the literature consists of pressure distributions alone, with little or no flow visualization data from which L_{sep} and α_{sep} can be measured. To resolve this problem the GE data were first analyzed to determine how accurately separation lengths could be calculated from pressure distribution data alone.

We take the "dead-air" region between the separating streamline and the corner as a wedge with an angle α_{sep}. Knowing M_0, p_0, and p_2, we compute α_{sep} by standard methods[9] as the wedge angle required to increase the pressure of an inviscid flow at M_0, from p_0 to p_2. The separated length is given by

$$L_{sep}' = (x_h - x_s)/[\cos\alpha_{sep} - (\sin\alpha_{sep}/\tan\delta_f)] \qquad (1)$$

Figure 4 compares L_{sep}' and L_{sep} for the GE data. Satisfactory agreement is obtained with a mean value of $L_{sep}' = 0.94\ L_{sep}$ and a total scatter of about ($\pm 0.075\ L_{sep}$). Thus from pressure distribution data alone, it is possible to compute the length from separation to reattachment of a separated flow region idealized to a wedge.

Data Analysis

Needham and Stollery correlated their transitional separation data by plotting $L_{sep}M_0{}^3/x_0\,(p_3/p_2)^2$ vs Re_{x_0}, the Reynolds number at the beginning of the separation interaction. The present data were found to correlate equally well using the simpler parameter $L_{sep}'\ M_0{}^3/x_0(p_f/p_0)^2$ where L_{sep}' is the separation length calculated from the plateau pressure and p_f/p_0 is the flap pressure ratio for inviscid flow (Fig. 5). The

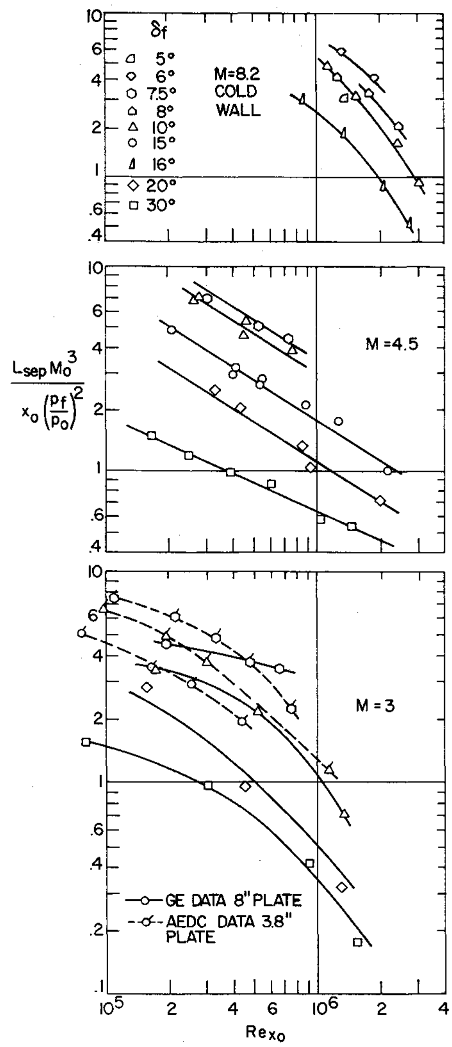

Fig. 5 Separated length correlations.

AEDC data of J. D. Gray,[8] at Mach 3, and Needham's Mach 8.2 data[10] also correlate in this way (Fig. 5).

The two sets of Mach 3 data (GE and AEDC) were taken with substantially the same model, in the same wind tunnel under the same range of flow conditions. Only x_h differed. However, when the two sets of data were plotted together a surprising discontinuity appeared. To explore this inconsistency the influence of the separation location x_s on the separated length L_{sep}' for the GE and AEDC data taken at the one common unit Reynolds number of 9.5×10^4 per inch, was examined (Fig. 6). It can be seen that under identical flow conditions x_s depends on the plate length. However, L_{sep}' is almost independent of x_s. As $x_0 \simeq x_s$, Re_{x0} cannot correlate the data using the two different configurations (plate lengths). The Needham-Stollery correlation is thus really a correlation against $(Re/\text{in.})_0$ for a single configuration. Figure 7 shows that $(Re/\text{in.})_0$ does indeed correlate the Mach 3 and 4.5 data quite well, as well as some Mach 5.5, 6.6, and 7.7. data of Kaufman.[11]† The curves in Fig. 7 are simply faired curves through the Mach 3 and 4.5 data, with the same functional form used for the higher Mach number data.

To understand why the separated length is independent of Re_{xs}, but dependent on $(Re/\text{in.})$, we consider the equation of

† Although Ref. 11 presents a large amount of transitional separation data, most of it could not be used because the data either showed a strong transverse pressure gradient, or the reattachment pressure on the flap was significantly lower than inviscid wedge pressure, apparently because the flap chord was too small. Either of these effects would decrease L_{sep}'.

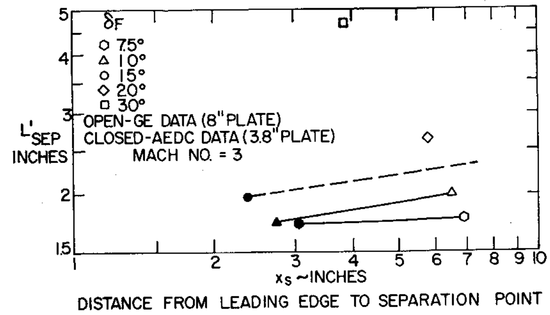

Fig. 6 Dependence of separated length on location of separation. Unit Reynolds number = 9.5 × 10⁶ per inch.

motion for the dividing streamline. A boundary layer separates from a surface when it encounters an adverse pressure gradient that is strong enough to reduce the normal velocity gradient at the wall $(\partial u/\partial y)_w$, to zero before the end of the pressure rise. When the flow separates, the dividing streamline (Fig. 1) leaves the wall and is accelerated by the outer inviscid flow acting in shear across the separated boundary layer. This acceleration takes place at almost constant static pressure (the plateau pressure), and continues until the dividing streamline acquires sufficient kinetic energy $(\rho u^2/2)$ to overcome the adverse reattachment pressure gradient, which is a function of inviscid flap pressure p_f. Thus, the two factors that determine L_{sep}' are p_f/p_0 and the effective viscosity in the boundary layer, that accelerates the dividing streamline. As the dividing streamline is almost a straight line (Fig. 1), we take an x' axis parallel to it and a y' axis normal to it. The boundary-layer momentum equation then applies along the dividing streamline

$$\rho(u'\partial u'/\partial x + v'\partial u'/\partial y') = -dp/dx' + \partial\tau/\partial y \qquad (2)$$

We can simplify and integrate Eq. (2) by noting that for an almost straight streamline $v' \simeq 0$ ρ is almost constant along the dividing streamline, and up to the beginning of reattachment $dp/dx \simeq 0$. We obtain

$$\frac{\rho u_{\max}'^2}{2} = \int_0^{L'} \frac{\partial\tau}{\partial y'}\, dx' \qquad (3)$$

where $u_{\max}'$ = maximum dividing streamline velocity required by $(\Delta p)_r$, $(\Delta p)_r$ = pressure rise from plateau pressure to reattachment, and L' = length of dividing streamline from seapration to beginning of reattachment pressure rise. For the shear stress we have $\tau = \rho\zeta(\partial u'/\partial y')$ where the effective viscosity $\zeta = \nu$, the kinematic viscosity for laminar flow, or ϵ, the eddy viscosity for turbulent flow, which is one to two orders of magnitude larger than ν.[12] Applying Bernoulli's equation, $\Delta p_r = \rho u_{\max}'^2/2$, and assuming $\partial(\rho\zeta)/\partial y' \simeq 0$ along the dividing streamline we have

$$\partial\tau/\partial y' \simeq \rho\zeta(\partial^2 u'/\partial y'^2) \qquad (4)$$

As x increases from zero $(\partial^2 u'/\partial y'^2)$ will decrease towards zero

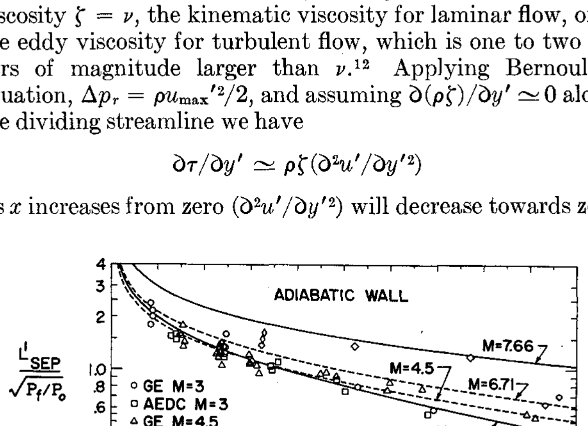

Fig. 7 Transitional boundary-layer separated length correlation against unit Reynolds number.

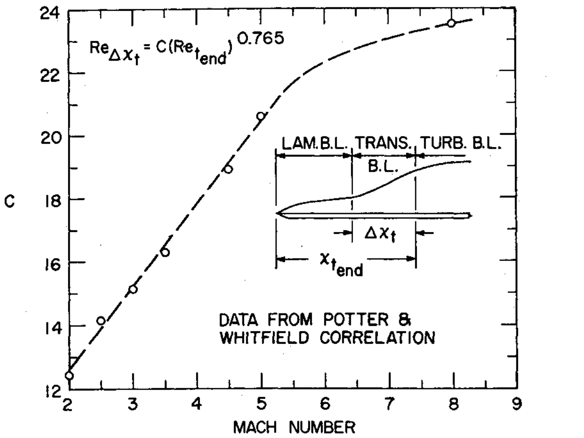

Fig. 8 Flat-plate transition length as a function of Mach number.

from some positive value at separation, while ζ, initially $\simeq \nu$, increases to $\simeq \epsilon$ with the growth of turbulence. Express $\partial^2 u'/\partial y'^2$ and ζ as power series expansions in (x/L): $\partial^2 u'/\partial y'^2 = a_0 + a_1(x'/L') + a_2(x'/L')^2 + \ldots$; and $\zeta = \zeta_L[b_0 + b_1(x'/L') + b_2(x'/L')^2 + \ldots]$, with ζ_L = value of ζ at $x' = L'$. Substituting and integrating,

$$(\Delta p)_r = \int_0^{L'} \rho \zeta_L \cdot \sum_n a_n \left(\frac{x'}{L'}\right)^n \cdot \sum_m b_m \left(\frac{x'}{L'}\right)^m dx' \quad (5)$$

or

$$(\Delta p)_r = \rho \zeta_L D L', \quad D = f(a_n, b_m) \quad (6)$$

The density along the dividing streamline is approximately $\rho = p_2/RT_w$ (where T_w = wall temperature), which yields $L'\zeta_L = [(\Delta p)_r/p_2]RT_w/D$.

To evaluate ζ we assume that it is related to the turbulent mixing that develops in the transition region of a flat-plate boundary layer with same freestream conditions. It might be thought that the development of transition in the separated region is related more closely to transition in a free-shear layer by analogy with turbulent separation, in which the separated region is analyzed as a turbulent jet.[13] In this case the wall would have no effect on transition development. The presence of the wall, however, is necessary for a steady-state transitional separation. Consider the recirculating flow between the dividing streamline and the wall (Fig. 1). In the forward flowing portion near the dividing streamline the turbulence increases from separation to reattachment. After reattachment the reversed flow becomes less turbulent as it moves towards the separation point as a result of both the favorable pressure gradient and the presence of the wall. It is reasonable to assume that the effect of the wall is also felt by the forward flowing fluid undergoing transition, in which case the transition

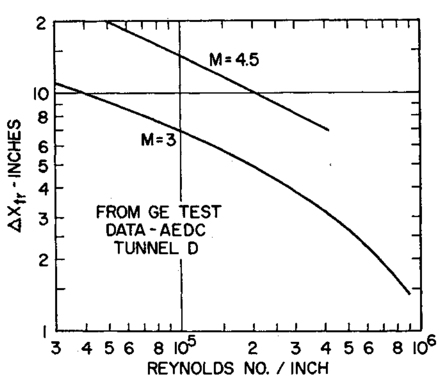

Fig. 9 Transition length as a function of freestream unit Reynolds number.

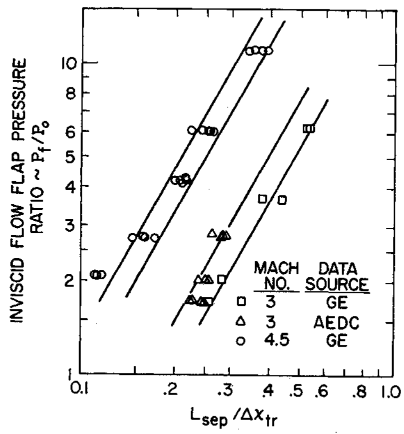

Fig. 10 Separated length as a function of flap pressure ratio.

process will be more akin to boundary-layer transition than free-shear layer transition.

The length of transition on a sharp leading edge flat plate Δx_{tr} has been correlated against the plate length to the end of transition $(x_t)_{end}$ by Potter and Whitfield,[14] for flows up to Mach 8. Their correlation can be simplified to that shown in Fig. 8. Transition length was measured as part of the GE experimental program at Mach 3 and 4.5. It agreed well with Potter and Whitfield's correlation, although at Mach 4.5 the onset of transition occurred somewhat farther downstream of the plate leading edge. Potter and Whitfield showed also that the beginning and end of transition depended strongly on $(Re/\text{in.})_0$, and this was confirmed in the GE data (Fig. 9). We thus find that $\Delta x_{tr} \sim (x_t)_{end} \sim (1/Re/\text{in.})_0$. This decrease in Δx_{tr} with the increase in $(Re/\text{in.})_0$ is caused by an increase in turbulent mixing and effective viscosity, ζ_L. Thus $\Delta x_{tr} \sim 1/\zeta_L$ or $\zeta_L \sim (1/\Delta x_{tr}) \sim (Re/\text{in.})_0$, and we have $L'/[(\Delta p)_r/p_2] \sim 1/(Re/\text{in.})_0$, which is the trend shown in Fig. 7. A more useful form is $L'/\Delta x_{tr} = [(\Delta p)_r/p_2]RT_w/D$. Examination of the data showed that correlating $(L_{sep}'/\Delta x_{tr})$ against p_f/p_0 yielded the best results. This is convenient as it eliminates the dependence on p_2.

The Δx_{tr} data from the GE tests[5,7] was used with L_{sep}' from both GE and AEDC data (Fig. 10). The correlation shown is $p_f/p_0 \sim (L_{sep}'/\Delta x_{tr})^2$. Plotting $L_{sep}'/\Delta x_{tr}(p_f/p_0)^{1/2}$ against M_0 we find a $M_0^{3/2}$ variation. Recombining our variables we obtain our final correlation (Fig. 11)

$$L_{sep}'M_0^{3/2}/\Delta x_{tr} \sim (p_f/p_0)^{1/2} \quad (7)$$

Data scatter is $\pm 12\%$ about the mean line.

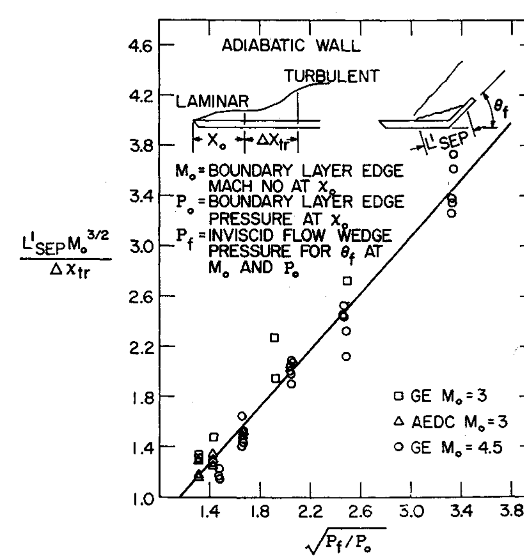

Fig. 11 Length of separated region for transitional separation upstream of a compression corner.

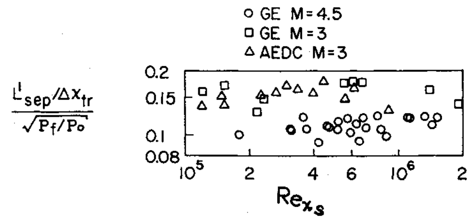

Fig. 12 Variation of correlating parameter with Reynolds number at the separation point.

It was thought that some of the scatter might be caused by a weak variation with Re_{x_s}, but Fig. 12 shows this is not so, confirming our previous conclusion based on Fig. 8.

Another possible source of the scatter is the inclusion of the reattachment zone as well as the constant pressure pleateau, in the separated length L_{sep}'. A correlation using only the length of the constant pressure portion of the dividing streamline showed as much scatter as one based on L_{sep}', indicating that this was not a significant error.

The remaining possible sources of scatter in Fig. 1[1] are inaccuracies in determining x_s and p_2/p_0. There is, first, an inherent variation in x_s introduced by substantial irregularities in the separation line, even for nominally two-dimensional flows. These appear consistently in surface oil flow studies made in regions of boundary-layer separation, such as those of Gray.[8] To this must be added an error introduced by estimating the separation point from a faired line through the surface static pressure data along the plate centerline. This latter error would also affect p_2. To evaluate these effects, the errors in x_s were estimated to be $\pm$ 0.10 in. for the GE data using the 8 in. plate, and $\pm$ 0.05 in. for the AEDC data using the 3.8 in. plate. The plateau pressure ratios, p_2/p_0, were taken as accurate to $\pm$ 0.06 and $\pm$0.03 respectively. The resulting average errors in L_{sep}' were $\pm$ 10.5% for the Mach 3 data and $\pm$ 7.3% for the Mach 4.5 data, comparable to the observed scatter. It is likely, therefore, that a substantial part of the scatter in the correlation arises from these sources.

Concluding Remarks

The transitional separation length L_{sep}' appears to be a simple function of Mach number M_0, flat-plate boundary-layer transition length Δx_{tr}, and inviscid flow flap pressure ratio p_f/p_0 at M_0 and flap angle δ_f. The dominant influence of the transition process explains why L_{sep}' is independent of the separation point location x_s or the Reynolds number based on x_s, Re_{x_s}, but is a function of unit Reynolds number $(Re/\text{in.})_0$.

The adverse pressure gradient at separation acts as a boundary-layer trip to induce early transition. This effect is used in two-dimensional boundary-layer trips, (wires or steps) which act by causing separation and reattachment. For a sufficiently large trip, transition will begin immediately downstream of the trip.[14] For most of the data used here, the separation process is a strong enough trip to initiate transition immediately. The ensuing growth of turbulent mixing, which determines the extent of separation, depends on $(Re/\text{in.})_0$, rather than x_s or Re_{x_s}.

Relating L_{sep}' to Δx_{tr} offers a way of scaling the transitional separation observed in ground test facilities to full-scale flight. The relation between boundary-layer transition observed in wind tunnels and unit Reynolds number is still not completely understood. It appears to be dependent on the particular wind tunnel being used. The ratio, $L_{sep}'/\Delta x_{tr}$, should be independent of the test facility and directly applicable to full scale vehicles. Flight data and further testing in different facilities is needed to verify and extend the correlation.

It is clear that an additional requirement, neglected in previous studies of transitional separation, is that *concurrent measurements must be made of the natural transition in the undisturbed flow, in order to correlate the results.* This is true regardless of the cause of separation; whether it be a control flap, forward facing step, impinging shock or other disturbance. In the present study extensive published transition data were available for the facility used. These data were verified by direct measurement of transition location, as part of the test program.

To extend these results, experimental work over a wider Mach number range is needed. Subsequently, wall cooling, and blowing or suction at the wall should be included. The present correlation should be able to include these wall effects with only minor modification.

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